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Robust Fault Detection and Isolation Using Robust ℓ_1 Estimation

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The application of robust ℓ_1 estimation to robust fault detection and isolation (FDI) is considered. This is accomplished by developing a series, or bank, of robust estimators (full-order observers), each of which is designed such that the residual will be sensitive to a certain fault (or faults) while insensitive to the remaining faults. Robustness is incorporated by ensuring that the residual remains insensitive to exogenous disturbances as well as modeling uncertainty. Mixed structured singular value and ℓ_1 theories are used to develop the appropriate threshold logic to evaluate the outputs of the estimators used for determining the occurrence and location of a fault. A real-coded genetic algorithm is used to obtain the estimator gain matrices. This approach to FDI is successfully demonstrated using a linearized model of a jet engine.

Nomenclature

$\mathcal{D}^n, \mathcal{N}^n, \mathcal{P}^n$	=	$n \times n$ real diagonal, nonnegative definite, positive definite matrices
$\mathrm{diag}(Z)$	=	$[z_{11}, z_{22}, \ldots, z_{nn}]^T$, $Z \in \mathcal{D}^n$
$\dim(M)$	=	dimension of M
$\|H_{zw}\|_1$	=	$\sup_{w(\cdot)\in\ell_\infty}(\|z\|_{\infty,2}/\|w\|_{\infty,2})$
$M_2 > M_1$	=	$M_2 - M_1$ positive definite
$M_2 \geq M_1$	=	$M_2 - M_1$ nonnegative definite
$\mathcal{R}, \mathcal{C}, \mathcal{Z}^+$	=	real numbers, complex numbers, nonnegative integers
$\mathcal{R}^{m\times n}, \mathcal{C}^{m\times n}$	=	$m \times n$ real matrices, complex matrices
$\mathrm{vec}(Z)$	=	$[z_{11}, \ldots, z_{m1}, z_{12}, \ldots, z_{m2}, \ldots, z_{mn}]^T$, $Z \in \mathcal{R}^{m\times n}$
z_{ij}	=	(i, j) element of matrix Z
$\|z(\cdot)\|_{\infty,2}$	=	$ess\,\sup_{t\geq 0}\|z(t)\|_2$
$\|z(\cdot)\|_{(\infty,2),[N_0,N]}$	=	$ess\,\sup_{t\in[N_0T,NT]}\|z(t)\|_2$
$0, I$	=	zero matrix, identity matrix

Introduction

IN modern systems such as aircraft and spacecraft, there is an increasing demand for reliability and safety. For example, a jet engine is very critical for an aircraft, and if faults occur, the consequences can be extremely serious.[1] Many dynamic systems are complex technical systems that involve extensive use of multiple sensors, actuators, and other system components, any one of which could fail or deteriorate. Hence, health monitoring and supervision of these systems is essential for the improvement of reliability, safety, and dependability of operations. This entails continuously checking a physical system for faults and taking appropriate actions to maintain the operation in such situations. In particular, the objective is to detect and isolate failures or anomalies in the sensors, actuators, and components.

One of the primary approaches to model-based, fault detection and isolation (FDI) uses state or output estimators.[2–8] Detection of a fault is achieved by comparing the actual behavior of the plant to that expected on the basis of the model; deviations are indications of a fault (or disturbances, noise or modeling errors).[9] Fault isolation can be achieved by dedicating an estimator such that the residual is sensitive to only one particular fault. In particular, with reference to Fig. 1, a bank of estimators is used to generate residuals $r(t)$. These residuals are then analyzed by some appropriate logic for example, logic based on thresholds or fuzzy logic, which infers whether faults have occurred (fault detection) and where they have occurred (fault isolation).

In many approaches to the FDI problem, the robustness aspect is commonly introduced in relation to the fault detection.[10] The estimators shown in Fig. 1 may be designed in a variety of ways, for example, by using Kalman filter theory, that is, H_2 optimal estimation,[11–13] H_∞ theory,[14–16] or ℓ_1 theory.[17–19] Whichever method is used for designing the estimator, it will use an idealized mathematical description of the underlying plant. In practice, this model of the plant is never totally accurate, which can degrade the quality of the residuals produced by the estimators. The errors in the plant model may be either parametric or unstructured, for example, unmodeled dynamics. To reduce the degradation in the quality of the residuals on which the FDI process is based and, hence, to reduce the false alarm rate, it is imperative that the plant uncertainty be explicitly taken into account in the design of the estimators.

Until recent work,[11,14,18,20] the relatively nonconservative mixed structured singular value (MSSV) techniques[21–24] had not been applied to robust estimation, although more conservative techniques, based on the small-gain theorem or fixed quadratic Lyapunov functions, had been used (see Refs. 15, 16, and 25–27). Conservatism in robustness theory involves how the theory actually models the uncertainty. For example, even if the uncertainty is real and parametric, the small-gain theorem assumes that the uncertainty is complex and unstructured. Likewise, fixed quadratic Lyapunov function theory assumes that the uncertainty is arbitrarily time-varying. MSSV theory, which considers both parametric uncertainty and unmodeled dynamics, allows real parametric uncertainty to be treated as slowly time-varying, real parametric uncertainty, which is a much less conservative model. The reduced conservatism allows the estimators to be used for more accurate fault detection. Specifically, the fixed thresholds are smaller, allowing the detection of smaller faults. With more conservative theories, the thresholds are larger, causing some smaller faults to go undetected. Although the example in this paper focuses exclusively on sensor faults, the theory is developed to include actuator faults as well.

This paper considers the application of robust ℓ_1 estimation to fault robust FDI. This is accomplished by developing a series, or bank, of robust estimators (full-order observers), each of which is designed such that the residual will be sensitive to a certain fault (or faults) while insensitive to the remaining faults. Robustness is incorporated by ensuring that the residual remains insensitive to exogenous disturbances as well as modeling uncertainty. MSSV and ℓ_1 theories are used to develop the appropriate threshold logic to evaluate the outputs of the estimators used for determining the occurrence and location of a fault. A real-coded genetic algorithm is used to obtain the estimator gain matrices. The effectiveness of this

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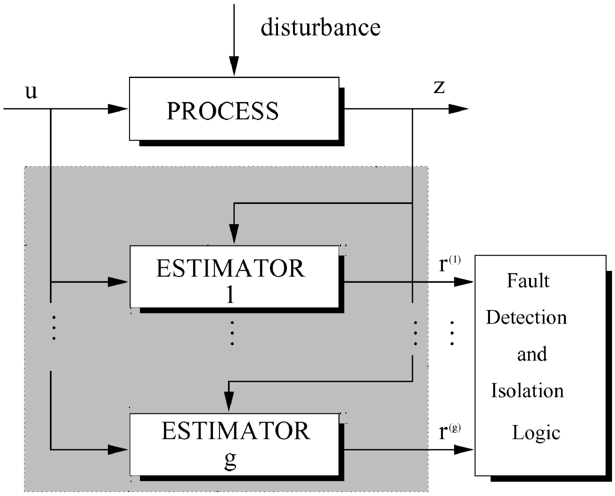


Fig. 1 Estimation-based FDI.

robust FDI technique is illustrated as it is applied to a discrete-time, linear uncertain model of an advanced afterburning turbofan engine.

The organization of this paper is as follows. In Sec. II, the formulation of the closed-loop uncertain system to which estimation will be applied is presented. The application of robust ℓ_1 estimation to robust fault detection and isolation is presented in Sec. III. In Sec. IV, results are discussed of an illustrative example of a jet engine, and in Sec. V, concluding remarks are made.

Robust ℓ_1 Estimation

Consider a discrete-time, linear uncertain dynamic system

$$x(k+1) = (A+\Delta A)x(k) + Bu(k) + D_{\infty,1}w_\infty(k), \qquad k \in \mathcal{Z}^+ \tag{1}$$

$$y(k) = (C+\Delta C)x(k) + Du(k) + D_{\infty,2}w_\infty(k) \tag{2}$$

$$z(k) = E_\infty x(k) \tag{3}$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^d$ is the control input, $y \in \mathcal{R}^p$ is the plant measurements, $z \in \mathcal{R}^q$ is the performance output to be estimated, and $w_\infty \in \mathcal{R}^{d_\infty}$ is an ℓ_∞ disturbance signal satisfying $\|w_\infty(\cdot)\|_{\infty,2} \le 1$. The uncertainties ΔA, ΔB, and ΔC satisfy

$$\Delta A \in \mathcal{U}_A \triangleq \left\{\Delta A \in \mathcal{R}^{n\times n} : \Delta A = -H_A F_A G_A,\ F_A \in \mathcal{F}_A\right\} \tag{4}$$

$$\Delta C \in \mathcal{U}_C \triangleq \left\{\Delta C \in \mathcal{R}^{p\times n} : \Delta C = -H_C F_C G_C,\ F_C \in \mathcal{F}_C\right\} \tag{5}$$

where

$$\mathcal{F}_A \triangleq \left\{F_A \in \mathcal{D}^r : M_{A,1} \le F_A \le M_{A,2}\right\} \tag{6}$$

$$\mathcal{F}_C \triangleq \left\{F_C \in \mathcal{D}^t : M_{C,1} \le F_C \le M_{C,2}\right\} \tag{7}$$

with $M_{A,1}$, $M_{A,2} \in \mathcal{D}^r$, $M_{C,1}$, $M_{C,2} \in \mathcal{D}^t$, $M_{A,2} - M_{A,1} \ge 0$, and $M_{C,2} - M_{C,1} \ge 0$.

It is desired to design a full-order observer of the form

$$x_e(k+1|k) = A_e x_e(k|k-1) + (B - WD)u(k) + W[y(k) - Cx_e(k|k-1)] \tag{8}$$

to estimate the state vector x, where $W \in \mathcal{R}^{n\times p}$ and $A_e \in \mathcal{R}^{n\times n}$ are the parameters to be determined.

The state estimation error is defined as

$$e(k) \triangleq x(k) - x_e(k|k-1) \tag{9}$$

which using Eqs. (1), (2), and (8) can be shown to obey the evolution equation

$$e(k+1) = (A_e - WC)e(k) + (A - A_e + \Delta A - W\Delta C)x(k) + (D_{\infty,1} - WD_{\infty,2})w_\infty(k) \tag{10}$$

Now define the error output $\tilde{z} \in \mathcal{R}^{q_p}$ as $\tilde{z}(k) \triangleq E_\infty e(k)$. Then augmenting Eq. (1) with Eq. (10) yields

$$\tilde{x}(k+1) = (\tilde{A} + \Delta\tilde{A})\tilde{x}(k) + \tilde{D}_1 w_\infty(k) \tag{11}$$

$$\tilde{z}(k) = \tilde{E}\tilde{x}(k) \tag{12}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} A & 0 \\ A - A_e & A_e - WC \end{bmatrix}$$

$$\tilde{D}_1 = \begin{bmatrix} D_{\infty,1} \\ D_{\infty,1} - WD_{\infty,2} \end{bmatrix}, \qquad \tilde{E} = [0 \quad E] \tag{13}$$

Furthermore, $\Delta\tilde{A}$ satisfies

$$\Delta\tilde{A} \in \tilde{\mathcal{U}}_A \triangleq \left\{\Delta\tilde{A} \in \mathcal{R}^{2n\times 2n} : \Delta\tilde{A} = -\tilde{H}_A\tilde{F}_A\tilde{G}_A,\ \tilde{F}_A \in \tilde{\mathcal{F}}_A\right\} \tag{14}$$

$$\tilde{\mathcal{F}}_A \triangleq \left\{\tilde{F}_A \in \mathcal{D}^{r+t} : M_1 \le \tilde{F}_A \le M_2\right\} \tag{15}$$

where

$$\tilde{F}_A = \begin{bmatrix} F_A & 0 \\ 0 & F_C \end{bmatrix}, \qquad \tilde{H}_A = \begin{bmatrix} H_A & 0 \\ H_A & -WH_C \end{bmatrix}$$

$$\tilde{G}_A = \begin{bmatrix} G_A & 0 \\ G_C & 0 \end{bmatrix} \tag{16}$$

$$M_1 = \mathrm{diag}(M_{A,1}, M_{C,1}), \qquad M_2 = \mathrm{diag}(M_{A,2}, M_{C,2}) \tag{17}$$

The robust ℓ_1 estimation problem is to find the estimator parameters A_e and W such that the combined system (8), (11), and (12) is asymptotically stable and the cost functional

$$\mathcal{J}(W) = \|H_{zw}\|_1^2 \tag{18}$$

is minimized, where H_{zw} is the the convolution operator from the disturbance $w_\infty(\cdot)$ to the ℓ_∞ performance variable $\tilde{z}(\cdot)$.

As shown in Ref. 28, direct minimization of the ℓ_1 norm can lead to irrational estimators. However, Haddad and Chellaboina[29] show it is possible to characterize an upper bound on the ℓ_1 performance. For some uncertainty set $\mathcal{U} \subset \mathcal{R}^{n\times n}$, $\Delta A \in \mathcal{U}$, $x \in \mathcal{R}^n$, $z \in \mathcal{R}^q$ $w_\infty(\cdot) \in \mathcal{R}^{d_\infty}$, the ℓ_1 performance bound as a function of ΔA is given in the following proposition.

Proposition: Let $\alpha > 1$ and assume there exists a positive-definite matrix $Q_{\Delta A}$ satisfying

$$Q_{\Delta\tilde{A}} = \alpha(\tilde{A} + \Delta\tilde{A})Q_{\Delta\tilde{A}}(\tilde{A} + \Delta\tilde{A})^T + [\alpha/(\alpha-1)]V_\infty \tag{19}$$

where $V_\infty \triangleq \tilde{D}_1\tilde{D}_1^T$. Then $\tilde{A} + \Delta\tilde{A}$ is asymptotically stable. Furthermore, the ℓ_1 norm of the convolution operator H_{zw} from disturbances $w(\cdot)$ to the performance variable $\tilde{z}(\cdot)$ satisfies the bound

$$\|H_{zw}\|_1^2 \le \sup_{\Delta\tilde{A} \in \mathcal{U}} \left[\mathrm{tr}\left(\tilde{E}Q_{\Delta\tilde{A}}\tilde{E}^T\right)^q\right]^{1/q}, \qquad \Delta\tilde{A} \in \mathcal{U} \tag{20}$$

If, in addition, α is chosen such that $\sqrt{\alpha}(\tilde{A} + \Delta\tilde{A})$ is asymptotically stable, then the existence of a positive-definite solution $Q_{\Delta\tilde{A}}$ is guaranteed.

Proof: The proof follows from the lemma in Ref. 18.

Remark: Minimization of the upper bound is a more appropriate approach than some conventional methods. Typical linear programming methods[30] that seek to minimize directly the ℓ_1 norm do not allow a fixed architecture estimator or controller design. These methods normally result in very high-order estimators or controllers, which are not practical for implementation.

To obtain an upper bound on the ℓ_1 performance $\|H_{zw}\|_1^2$ over the entire uncertainty set $\mathcal{U}$, a multiplier framework will be used to bound $Q_{\Delta\tilde{A}}$ for all $\Delta A \in \mathcal{U}$, where

$$\mathcal{U} \triangleq \left\{\Delta\tilde{A} \in \mathcal{R}^{2n\times 2n} : \Delta\tilde{A} = -H_0 F G_0,\ F \in \mathcal{F}\right\} \tag{21}$$

Let $G(z) \in \mathcal{C}^{q\times d_\infty}$ be the transfer function representation of the system described in system (11) and (12). The Popov–Tsypkin multiplier (see Refs. 11, 14, 18, and 24) has the transfer function form

$$M(z) = H + N[(z-1)/z] \tag{22}$$

where $H \in \mathcal{D}^m$ and $N \in \mathcal{D}^m (m = r + t)$ with $H > 0$ and $N \geq 0$. Let A_a be the state matrix of the augmented system $M(z)G(z)$. Then, the uncertain system for robust analysis is given by[31]

$$\boldsymbol{x}_a(k+1) = (A_a + \Delta A_a)\boldsymbol{x}_a(k) + D_{a,\infty}\boldsymbol{w}_\infty(k) \tag{23}$$

$$z(k) = E_a\boldsymbol{x}_a(k) \tag{24}$$

where $\boldsymbol{x}_a(k) = [\boldsymbol{x}_m^T(k)\boldsymbol{x}^T(k)]^T$, $\boldsymbol{x}_m(k) \in \mathcal{R}^m$ denotes the states of the multiplier,

$$A_a = \begin{bmatrix} 0 & 0 \\ H_0 & \tilde{A} \end{bmatrix}, \qquad D_{a,\infty} = \begin{bmatrix} 0 \\ D_\infty \end{bmatrix}, \qquad E_a = [0 \quad E] \tag{25}$$

$$\Delta A_a \in \mathcal{U}_a \triangleq \left\{\Delta A_a \in \mathcal{R}^{m+n} : \Delta A_a = -H_a F G_a,\ F \in \mathcal{F}\right\} \tag{26}$$

where

$$H_a = \begin{bmatrix} 0 \\ H_0 \end{bmatrix}, \qquad G_a = [0 \quad G_0] \tag{27}$$

Note that the uncertainty set $\mathcal{U}$ in Eq. (21) is a subset of $\mathcal{U}_a$. The next theorem provides an upper bound for the ℓ_1 performance for all $\Delta A_a \in \mathcal{U}_a$.

Theorem: Let $\alpha > 1$, $q \geq 1$. Suppose there exists $H \in \mathcal{P}^n$, $N \in \mathcal{N}^n$, and $Q_a \geq 0$ such that $2H(M_2 - M_1)^{-1} - G_a Q_a G_a^T > 0$, and Q_a satisfies the algebraic Riccati equation

$$\begin{aligned} Q_a &= \alpha(A_a - H_a M_1 G_a)Q_a(A_a - H_a M_1 G_a)^T \\ &\quad + \left[\sqrt{\alpha}(A_a - H_a M_1 G_a)Q_a C_a^T - \sqrt{\alpha}B_a(H+N) + S_a N\right] \\ &\quad \times \left[2H(M_2 - M_1)^{-1} - G_a Q_a G_a^T\right]^{-1}\left[\sqrt{\alpha}(A_a - H_a M_1 G_a)\right. \\ &\quad \left.\times Q_a C_a^T - \sqrt{\alpha}B_a(H+N) + S_a N\right]^T + [\alpha/(\alpha-1)]V_{a,\infty} \end{aligned} \tag{28}$$

where

$$V_{a,\infty} \triangleq D_{a,\infty}D_{a,\infty}^T, \qquad S_a \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix}$$

where $\dim(S_a) = \dim(H_a)$.

Then

$$\left((A_a + \Delta A_a), \{[\alpha/(\alpha-1)]V_{a,\infty}\}^{\frac{1}{2}}\right) \text{is stabilizable}, \ \Delta A \in \mathcal{U} \tag{29}$$

if and only if $(A_a + \Delta A_a)$ is asymptotically stable for each $\Delta A_a \in \mathcal{U}_a$ and in this case, the ℓ_1 performance $\|H_{zw}\|_1^2$ is bounded as

$$\|H_{zw}\|_1^2 \leq \left[\mathrm{tr}\left(E_a Q_a E_a^T\right)^q\right]^{1/q}, \qquad \Delta A_a \in \mathcal{U}_a \tag{30}$$

Proof: The proof can be completed in the same manner as the proof to results in Ref. 23.

Robust FDI Using Robust ℓ_1 Estimation

Now consider the fault-driven system

$$\boldsymbol{x}(k+1) = (A + \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) + D_{\infty,1}\boldsymbol{w}_\infty(k) + R_a\boldsymbol{f}_a(k) \tag{31}$$

$$\boldsymbol{y}(k) = (C + \Delta C)\boldsymbol{x}(k) + D_p\boldsymbol{u}(k) + D_{\infty,2}\boldsymbol{w}_\infty(k) + R_s\boldsymbol{f}_s(k) \tag{32}$$

$$z(k) = E_\infty\boldsymbol{x}(k) \tag{33}$$

where $\boldsymbol{f}_a \in \mathcal{R}^{r_a}$ and $\boldsymbol{f}_s \in \mathcal{R}^{r_s}$ are the actuator and sensor fault vectors, respectively. The fault distribution matrices R_a and R_s are assumed to be known. Defining $\boldsymbol{f} \triangleq [\boldsymbol{f}_a^T \ \boldsymbol{f}_s^T]^T$ yields the modified system

$$\boldsymbol{x}(k+1) = (A + \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) + D_{\infty,1}\boldsymbol{w}_\infty(k) + R_1\boldsymbol{f}(k) \tag{34}$$

$$\boldsymbol{y}(k) = (C + \Delta C)\boldsymbol{x}(k) + D_{\infty,2}\boldsymbol{w}_\infty(k) + R_2\boldsymbol{f}(k) \tag{35}$$

$$z(k) = E_\infty\boldsymbol{x}(k) \tag{36}$$

where

$$R_1 = [R_a \quad 0], \qquad R_2 = [0 \quad R_s] \tag{37}$$

Equations (34) and (35) can be written in the expanded forms

$$\begin{aligned} \boldsymbol{x}(k+1) &= (A + \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) + D_{\infty,1}\boldsymbol{w}_\infty(k) \\ &\quad + R_{1,1}\boldsymbol{f}_1(k) + \cdots + R_{1,g}\boldsymbol{f}_g(k) \end{aligned} \tag{38}$$

$$\begin{aligned} \boldsymbol{y}(k) &= (C + \Delta C)\boldsymbol{x}(k) + D_{\infty,2}\boldsymbol{w}_\infty(k) + R_{2,1}\boldsymbol{f}_1(k) \\ &\quad + \cdots + R_{2,g}\boldsymbol{f}_g(k) \end{aligned} \tag{39}$$

where $R_{1,i}$ (respectively, $R_{2,i}$) is the ith column of the matrix R_1 (respectively, R_2). Let $g \triangleq r_a + r_s$. For $i \in \{1, 2, \ldots, g\}$, the term $\boldsymbol{f}_i(k)$ is the ith individual fault of $\boldsymbol{f}(k)$, and $R_{1,i}$ (respectively, $R_{2,i}$) is its directional characteristics. Assume that $\boldsymbol{f}_i(k)$ is the target fault, that is, the fault that it is desired to detect. Without loss of generality, the vector of nuisance faults, representing the faults that are not to be detected (by the robust fault detection filter), is given by $\bar{\boldsymbol{f}}_i \triangleq [\boldsymbol{f}_1(k)\cdots\boldsymbol{f}_{i-1}(k)\boldsymbol{f}_{i+1}(k)\cdots\boldsymbol{f}_g(k)]$. Hence, Eqs. (38) and (39) can be replaced by

$$\begin{aligned} \boldsymbol{x}(k+1) &= (A + \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) + D_{\infty,1}\boldsymbol{w}_\infty(k) \\ &\quad + R_{1,i}\boldsymbol{f}_i(k) + \bar{R}_{1,i}\bar{\boldsymbol{f}}_i(k) \end{aligned} \tag{40}$$

$$\boldsymbol{y}(k) = (C + \Delta C)\boldsymbol{x}(k) + D_{\infty,2}\boldsymbol{w}_\infty(k) + R_{2,i}\boldsymbol{f}_i(k) + \bar{R}_{2,i}\bar{\boldsymbol{f}}_i(k) \tag{41}$$

Define $\tilde{\boldsymbol{w}} \triangleq [\boldsymbol{w}^T \ \bar{\boldsymbol{f}}_i^T]^T$. Then, Eqs. (40) and (41) can be written as a set of systems

$$\Sigma_i \begin{cases} \boldsymbol{x}(k+1) = (A + \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) + D_1\tilde{\boldsymbol{w}}(k) + R_{1,i}\boldsymbol{f}_i(k) \\ \boldsymbol{y}(k) = C\boldsymbol{x}(k) + D_2\tilde{\boldsymbol{w}}(k) + R_{2,i}\boldsymbol{f}_i(k) \\ z(k) = E_\infty\boldsymbol{x}(k) \end{cases} \tag{42}$$

where

$$D_1 = [D_{\infty,1} \quad \bar{R}_{1,i}], \qquad D_2 = [D_{\infty,2} \quad \bar{R}_{2,i}] \tag{43}$$

It is desired to design a bank of full-order observers (corresponding to each faulty system) described in Eq. (8) to estimate the performance output $E_\infty\boldsymbol{x}(k)$. As stated earlier, $A_e \in \mathcal{R}^{n\times n}$ and $W \in \mathcal{R}^{n\times p}$ are the parameters to be determined. (Typically, one chooses $E_\infty = C$ such that z corresponds to the noise and fault-free output associated with the measurement $\boldsymbol{y}$.) Detection of a fault is achieved by comparing the actual behavior of the plant to the the output of the estimators; deviations are indications of a fault (or disturbances, noise, or modeling errors).

Let the residual error be defined as

$$r(k) \triangleq P[\boldsymbol{y}(k) - C\boldsymbol{x}_e(k|k-1) - D\boldsymbol{u}(k)] \tag{44}$$

where the $g \times p$ gain matrix P (Ref. 32) is chosen such that r has a fixed direction when responding to the target fault. Fault isolation can be achieved by designing an estimator such that estimation error, that is, the residual, is sensitive to only one particular fault. Specifically, each is designed to be sensitive to a particular fault and insensitive to the remaining faults. In addition, these estimators are made robust against exogenous disturbances and modeling uncertainties. With reference to Fig. 1, the bank of estimators is used to generate residuals $r(k)$. These residuals are then analyzed by some appropriate logic, for example, logic based on thresholds, which infers whether faults have occurred (fault detection) and where they have occurred (fault isolation).

Using set (42), the state estimation error in Eq. (9) can be shown to obey the evolution equation

$$e(k+1) = (A_e - WC)e(k) + (A + \Delta A - W\Delta C - A_e)\boldsymbol{x}(k) + (D_1 - WD_2)\boldsymbol{w}_\infty(k) + (R_{1,i} - WR_{2,i})\boldsymbol{f}_i(k) \tag{45}$$

Augmenting set (42) with Eq. (45) yields

$$\tilde{\boldsymbol{x}}(k+1) = (\tilde{A} + \Delta\tilde{A})\tilde{\boldsymbol{x}}(k) + \tilde{D}_1\boldsymbol{w}_\infty(k) + \tilde{R}_1\boldsymbol{f}_i(k) \tag{46}$$

$$\tilde{z}(k) = \tilde{E}\tilde{\boldsymbol{x}}(k) \tag{47}$$

where

$$\tilde{\boldsymbol{x}}(k) = \begin{bmatrix} \boldsymbol{x}(k) \\ e(k) \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} A & 0 \\ A - A_e & A_e - WC \end{bmatrix}$$

$$\tilde{D}_1 = \begin{bmatrix} D_1 \\ D_1 - WD_2 \end{bmatrix}, \qquad \tilde{R}_1 = \begin{bmatrix} R_{1,i} \\ R_{1,i} - WR_{2,i} \end{bmatrix}$$

$$\tilde{E} = [0 \quad E_\infty] \tag{48}$$

Let J_{rw} be the ℓ_1 norm of the system operator from the disturbance vector $\tilde{\boldsymbol{w}}$ to the residual r and let J_{rf} represent the ℓ_1 norm of the system operator from the target fault $\boldsymbol{f}_i$ to the residual r. When earlier derivations are followed, it is possible to characterize upper bounds $\mathcal{J}_{rw}$ and $\mathcal{J}_{rf}$ such that

$$J_{rw} = \|H_{rw}\|_1^2 \le \mathcal{J}_{rw} \tag{49}$$

$$J_{rf} = \|H_{rf}\|_1^2 \le \mathcal{J}_{rf} \tag{50}$$

When multiplier theory is used, the uncertain system is given by

$$\boldsymbol{x}_a(k+1) = (A_a + \Delta A_a)\boldsymbol{x}_a(k) + D_{a,w}\tilde{\boldsymbol{w}}(k) + D_{a,f}\boldsymbol{f}_i(k) \tag{51}$$

$$\tilde{z}(k) = E_a\boldsymbol{x}_a(k) \tag{52}$$

where $\boldsymbol{x}_a(k) = [\boldsymbol{x}_m^T(k) \;\; \tilde{\boldsymbol{x}}^T(k)]^T$ and $\boldsymbol{x}_m(k) \in \mathcal{R}^m$ is as already described. The ℓ_1 performance functions then have the bounds

$$\mathcal{J}_{rw}(A_e, W, P, H, N) = \left[\mathrm{tr}\left(E_a Q_{a,w} E_a^T\right)^q\right]^{1/q} \tag{53}$$

$$\mathcal{J}_{rf}(A_e, W, P, H, N) = \left[\mathrm{tr}\left(E_a Q_{a,f} E_a^T\right)^q\right]^{1/q}, \qquad \Delta A_a \in \mathcal{U}_a \tag{54}$$

where $Q_{a,w}$ and $Q_{a,f}$ satisfy the algebraic Riccati equations

$$\begin{aligned} Q_{a,w} &= \alpha(A_a - H_aM_1G_a)Q_{a,w}(A_a - H_aM_1G_a)^T \\ &\quad + \left[\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,w}C_a^T - \sqrt{\alpha}H_a(H+N) + S_aN\right] \\ &\quad \times \left[2H(M_2 - M_1)^{-1} - G_aQ_{a,w}G_a^T\right]^{-1} \\ &\quad \times \left[\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,w}C_a^T - \sqrt{\alpha}H_a(H+N) + S_aN\right]^T \\ &\quad + [\alpha/(\alpha-1)]V_{a,w} \end{aligned} \tag{55}$$

$$\begin{aligned} Q_{a,f} &= \alpha(A_a - H_aM_1G_a)Q_{a,f}(A_a - H_aM_1G_a)^T \\ &\quad + \left[\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,f}C_a^T - \sqrt{\alpha}H_a(H+N) + S_aN\right] \\ &\quad \times \left[2H(M_2 - M_1)^{-1} - G_aQ_{a,f}G_a^T\right]^{-1} \\ &\quad \times \left[\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,f}C_a^T - \sqrt{\alpha}H_a(H+N) + S_aN\right]^T \\ &\quad + [\alpha/(\alpha-1)]V_{a,f} \end{aligned} \tag{56}$$

where

$$V_{a,w} \triangleq D_{a,w}D_{a,w}^T, \qquad V_{a,f} \triangleq D_{a,f}D_{a,f}^T, \qquad S_a \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix}$$

with $\dim(S_a) = \dim(H_a)$.

Robust FDI filter design may be approached by choosing A_e, W, and P such that $\mathcal{J}_{rw}$ is small and $\mathcal{J}_{rf}$ is large. (It would be more desirable to make a lower bound on $\mathcal{J}_{rf}$ large. Unfortunately, lower bounds are usually much more difficult to work with computationally than upper bounds.) A minimization problem that expresses this objective is

$$\min_{A_e, W, P} \mathcal{J} = \beta\mathcal{J}_{rw} + (1-\beta)(1/\mathcal{J}_{rf}) + \gamma(\mathcal{J}_{rw}/\mathcal{J}_{rf}) \tag{57}$$

where $\beta \in [0, 1]$ and $\gamma > 0$ are arbitrarily chosen weighting scalars. With an enforced stability constraint, this optimization problem can be solved using a real-coded genetic algorithm, as discussed in the next section.

Now consider the set of uncertain, discrete-time systems

$$\boldsymbol{x}(k+1) = (A + \Delta A)\boldsymbol{x}(k) + D_1\boldsymbol{w}_\infty(k) + R_{1,i}\boldsymbol{f}_i(k) \tag{58}$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) + D_2\boldsymbol{w}_\infty(k) + R_{2,i}\boldsymbol{f}_i(k) \tag{59}$$

$$z(k) = E_\infty\boldsymbol{x}(k) \tag{60}$$

where $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{w}$, and $\boldsymbol{f}_i$ are as discussed earlier. The robust fault detection problem is to generate a set robust residual signals $r(k)$ that satisfy

$$\|r(k)\|_p \le J_{\mathrm{th}} \qquad \text{if} \qquad \boldsymbol{f}_i(k) = 0 \tag{61}$$

$$\|r(k)\|_p > J_{\mathrm{th}} \qquad \text{if} \qquad \boldsymbol{f}_i(k) \ne 0 \tag{62}$$

where $\|\cdot\|_p$ denotes the p norm of a Lebesgue signal and J_{th} is the ith threshold value. If the filters (8) are applied to Eq. (58) and (59) and E_∞ is chosen as C, Eq. (44) can be written as

$$r(k) = Pz(k) + PD_2\boldsymbol{w}_\infty(k) + PR_{2,i}\boldsymbol{f}_i(k) \tag{63}$$

As derived in Ref. 18, if $\boldsymbol{f}_i(k) = 0$ Eq. (63) satisfies the norm inequality

$$\begin{aligned} \|r\|^2_{(\infty,2),[N_0,N]} &\le \left\{\left[\mathrm{tr}\left(PE_aQ_{a,w}E_a^TP^T\right)^q\right]^{1/q} \right. \\ &\quad + 2\sigma_{\max}(PD_2)\left[\mathrm{tr}\left(PE_aQ_{a,w}E_a^TP^T\right)^q\right]^{1/2q} \\ &\quad \left. + \sigma^2_{\max}(PD_2)\right\}\|\tilde{\boldsymbol{w}}\|^2_{(\infty,2),[N_0,N]} \end{aligned} \tag{64}$$

where $Q_{a,w}$ is as defined earlier. The threshold can be chosen as

$$\begin{aligned} J_{\mathrm{th}} &\triangleq \left\{\left[\mathrm{tr}\left(PE_aQ_{a,w}E_a^TP^T\right)^q\right]^{1/q} \right. \\ &\quad + 2\sigma_{\max}(PD_2)\left[\mathrm{tr}\left(PE_aQ_{a,w}E_a^TP^T\right)^q\right]^{1/2q} \\ &\quad \left. + \sigma^2_{\max}(PD_2)\right\}\|\tilde{\boldsymbol{w}}\|^2_{(\infty,2),[N_0,N]} \end{aligned} \tag{65}$$

Robust fault detection can be accomplished by comparing $\|r\|_{(\infty,2),[N_0,N]}$ with J_{th}. A fault occurs if $\|r\|_{(\infty,2),[N_0,N]} > J_{\mathrm{th}}$, that is,

$$\|r\|_{(\infty,2),[N_0,N]} > J_{\mathrm{th}} \Rightarrow \text{a fault occurred} \tag{66}$$

Optimization Using a Real-Coded Genetic Algorithm

As discussed earlier, the design of robust FDI estimators is formulated as an optimization problem. A real coded genetic algorithm (RCGA) is used to search for a solution. Genetic algorithms (GAs) can efficiently search in complex and possibly discontinuous solution spaces without problem reformulation or evaluation of each solution candidate. They offer the following additional advantages over traditional methods: 1) Information about derivatives, Hessians, or step sizes is not required. 2) A population of points in the solution space is searched in parallel rather than point by point. 3) A number of potential solutions to a given problem can be provided. GAs have been proven to provide efficiency, that is, faster computation times and smaller storage, and flexibility, that is, adaptation to a range of complex problems, in comparison to traditional methods of optimization. The use of RCGAs, where operations are performed with real numbers, rather than binary GAs, where binary digits are used, proves to be more advantageous. Because no coding or decoding of binary numbers is necessary, a subsequent decrease in computational time and storage size is achieved.

An RCGA begins with an arbitrarily chosen initial population within the search region. The algorithm then follows three general operations: 1) selection, 2) recombination, and 3) mutation.[33–36] The flowchart for a single-population RCGA is shown in Fig. 2.

Selection

A common selection process in RCGAs is conducted using stochastic universal sampling.[35] Individuals of a population are mapped to a line segment, such that each individual's segment is equal in size to its normalized fitness value. Then, N equally spaced pointers are placed along the line segment, where N is the number of individuals to be selected. The position of the first pointer is determined by a randomly generated number $p \in [0, 1/N]$, where $1/N$ is the spacing between pointers. This method of selection is analogous to roulette wheel selection[34] and is shown in Fig. 3 for a population of eight individuals, n_i, with $N = 4$. From this example it can be seen that individuals n_2, n_3, n_5, and n_7 are chosen.

Recombination

In an RCGA, recombination is parallel to crossover in a binary GA. It is the process by which new chromosomes are produced from existing ones and involves the exchange of the individuals' numeric values (genes).[35,36] Let p_1 and p_2 represent two individuals (parents)

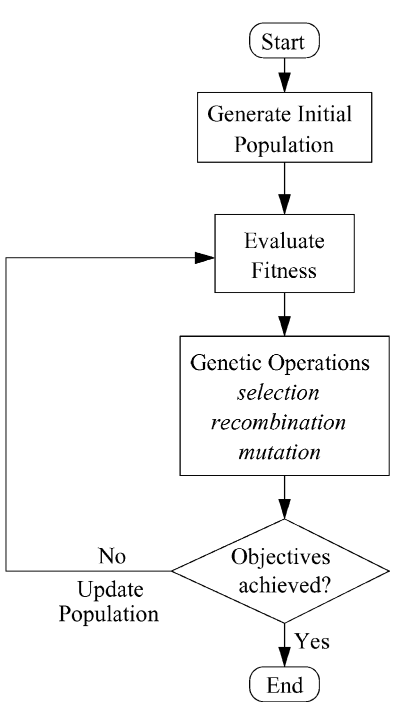


Fig. 2 Flowchart of single-population RCGA.

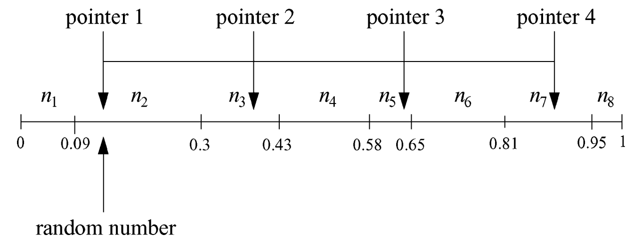


Fig. 3 Stochastic universal sampling for real-coded selection.

who are to reproduce. The offspring p_1' and p_2' are produced as a linear combination of the parents,

$$p_1' = \alpha p_1 + (1-\alpha) p_2 \tag{67}$$

$$p_2' = (1-\alpha) p_1 + \alpha p_2 \tag{68}$$

where $\alpha \in [0, 1]$ is a recombination parameter.

Mutation

The mutation process was originally developed for binary representation. However, other methods have been developed to allow gene modification in an RCGA. The mutation operation randomly alters one or more genes of a selected chromosome. More specifically, randomly generated values are added to the genes with low probability. The probability of mutation is inversely proportional to the number of variables (dimensions). The more dimensions an individual has, the smaller the mutation probability.[35] An effective mutation operator, which produces small step sizes with a high probability and large step sizes with a low probability, uniform at random, is defined as

$$Gen_i^{\text{mut}} = Gen_i + s_i r_i a_i, \qquad i \in \{1, 2, \ldots, m\} \tag{69}$$

$$s_i \in \{-1, +1\} \tag{70}$$

$$a_i = 2^{-uk}, \qquad u \in [0, 1] \tag{71}$$

where s, r, and a are direction, mutation range, and relative step size, respectively, and m is the number of genes in the chromosome. The mutation range is defined in terms of the domain of the genes, and the step size is defined in terms of the mutation precision k.

For the robust ℓ_1 optimization problem, the chromosome is constructed by formulating matrices A_e, W, P, H, and N into a single vector $\mathbf{\Theta}$ such that

$$\mathbf{\Theta} = \begin{bmatrix} \mathrm{vec}(\boldsymbol{A}_e)^T & \mathrm{vec}(\boldsymbol{W})^T & \mathrm{vec}(\boldsymbol{P})^T & \mathrm{diag}(H)^T & \mathrm{diag}(N)^T \end{bmatrix} \tag{72}$$

The search region is then defined by establishing upper and lower limits $\bar{\mathbf{\Theta}}$ and $\underline{\mathbf{\Theta}}$ such that

$$\underline{\theta}_{ij} \le \theta_{ij} \le \bar{\theta}_{ij} \tag{73}$$

To account for the stability criteria, the RCGA is formulated as a constrained optimization problem. This is achieved by imposing a constraint on the cost with a penalty function. Specifically, if the stability criterion is not satisfied, a multiplicative penalty is imposed on the cost such that

$$\text{if} \begin{cases} \max[\lambda_i(A_a)] < 1, & \mathcal{J} = \mathcal{J} \\ \text{otherwise}, & \mathcal{J} = \text{penalty} \times \mathcal{J} \end{cases} \tag{74}$$

where λ_i, $i \in (1, 2, \ldots, m + 2n)$, are the eigenvalues of the augmented system A_a. Using this type of penalty helps to ensure that, because of fitness values, individuals representing unstable systems will not survive the selection process. The penalty is chosen as 100 such that the unstable fitness values will be two orders of magnitude larger than their true values.

Illustrative Example of FDI for a Jet Engine

A numerical example is presented to illustrate robust ℓ_1 estimator design using the Popov–Tsypkin multiplier and the application of the robust ℓ_1 estimator to robust fault detection of dynamic systems. The model used was supplied by NASA John H. Glenn Research Center at Lewis Field and is given as

$$\boldsymbol{x}(k+1) = (A + \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) + D_{\infty,1}\boldsymbol{w}_{\infty}(k) + R_a \boldsymbol{f}_a(k) \tag{75}$$

$$\boldsymbol{y}(k) = (C + \Delta C)\boldsymbol{x}(k) + D\boldsymbol{u}(k) + D_{\infty,2}\boldsymbol{w}_{\infty}(k) + R_s \boldsymbol{f}_s(k) \tag{76}$$

where the sampling period $T_s = 0.01$ s. Only sensor faults are considered in this example; thus, $R_a = 0$. The elements of the state vector $\boldsymbol{x} \in \mathcal{R}^3$ are $\boldsymbol{x}_1 \triangleq$ high-pressure spool speed (revolutions per minute), $\boldsymbol{x}_2 \triangleq$ low-pressure spool speed (revolutions per minute), and $\boldsymbol{x}_3 \triangleq$ high-pressure compressor inlet temperature (degrees Celsius).

The elements of the control input vector $\boldsymbol{u} \in \mathcal{R}^3$ are $\boldsymbol{u}_1 \triangleq$ main burner fuel flow (kilograms per hour), $\boldsymbol{u}_2 \triangleq$ exhaust nozzle throat area (square meters), and $\boldsymbol{u}_3 \triangleq$ bypass duct area (square meters).

The elements of the output vector $\boldsymbol{y} \in \mathcal{R}^3$ are $\boldsymbol{y}_1 \triangleq$ corrected high-pressure spool speed (revolutions per minute), $\boldsymbol{y}_2 \triangleq$ corrected low-pressure spool speed (revolutions per minute), and $\boldsymbol{y}_3 \triangleq$ corrected high-pressure compressor inlet temperature (degrees Celsius).

The variable $\boldsymbol{w}$ denotes a vector of disturbance signals.

The uncertainty matrices, ΔA and ΔC, are representative of some engine degradation over time. Thus, it is assumed that a newly constructed engine can be modeled with the nominal matrices A and C and with use, the parameters of the degraded engine are encompassed in the uncertainty. The system parameter matrices are

$$A = \begin{bmatrix} 0.9835 & 0.0110 & 0.0039 \\ 3.788e{-4} & 0.9858 & 0.0026 \\ 4.230e{-6} & -2.282e{-4} & 0.9891 \end{bmatrix}$$

$$D_{\infty,1} = \mathrm{diag}\{0.1, 0.1, 0.01\} \tag{77}$$

$$B = \begin{bmatrix} 0.0080 & 0.2397 & -0.0383 \\ 0.0068 & 0.1565 & 0.0248 \\ 2.691e{-4} & -2.912e{-4} & 2.558e{-4} \end{bmatrix}$$

$$R_s = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{78}$$

$$C = \begin{bmatrix} 0.2383 & 0.4871 & 0.1390 \\ -1.074e{-5} & -8.399e{-4} & 3.784e{-4} \\ 2.070e{-5} & -4.132e{-5} & -4.335e{-6} \end{bmatrix} \tag{79}$$

$$D = \begin{bmatrix} 0.4171 & -4.492 & 0.4875 \\ 7.968e{-4} & -0.0050 & 2.861e{-4} \\ -1.270e{-5} & 4.837e{-4} & -0.0021 \end{bmatrix}$$

$$D_{\infty,2} = 0.1 \times I_{3\times 3} \tag{80}$$

The uncertainty matrices $\Delta A = -H_A F_A G_A$ and $\Delta C = -H_C F_C G_C$, where

$$H_A = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad H_C = -\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_A = [\, I_{3\times 3} \,], \qquad G_C = [\, I_{3\times 3} \,]$$

$$F_A = \mathrm{diag}\left\{\delta_{A_1}, \delta_{A_2}, \delta_{A_3}\right\}, \qquad F_C = \mathrm{diag}\left\{\delta_{C_1}, \delta_{C_2}, \delta_{C_3}\right\} \tag{81}$$

with

$$-0.02167 \le \delta_{A_1} \le 0.02167, \qquad -0.02174 \le \delta_{A_2} \le 0.02174$$
$$-0.02181 \le \delta_{A_3} \le 0.02181 \tag{82}$$

$$-0.01787 \le \delta_{C_1} \le 0.01787, \qquad -0.03653 \le \delta_{C_2} \le 0.03653$$
$$-0.01043 \le \delta_{C_3} \le 0.01043 \tag{83}$$

Note that the uncertain parameters $\delta_{A_1}, \ldots, \delta_{A_3}$ correspond to parameter fluctuations in the diagonal elements of matrix A and $\delta_{C_1}, \ldots, \delta_{C_3}$ correspond to the first row of C. In the RCGA, the chromosome string $\boldsymbol{\Theta}$ [Eq. (72)] consisted of 39 genes, corresponding to the elements of $A_e \in \mathcal{R}^3$, $W \in \mathcal{R}^3$, and $P \in \mathcal{R}^3$ and the diagonal elements of H, and $N \in \mathcal{D}^6$. When the objective function (57) is used with stability constraints (74), the respective gain and projection matrices are obtained for a bank of estimators. The nominal (uncertainty not considered) gain matrices were

$$W_{n,1} = \begin{bmatrix} -4.3677 & 0.9652 & 2.1343 \\ -46.331 & -0.4433 & 2.5012 \\ 185.10 & 0.0755 & -12.637 \end{bmatrix}$$

$$P_{n,1} = \begin{bmatrix} 0.0324 & -0.1080 & 1.7946 \\ -0.0042 & 0.0218 & -0.0179 \\ 0.0242 & -0.0931 & 1.3789 \end{bmatrix} \tag{84}$$

$$W_{n,2} = \begin{bmatrix} 8.5628 & 2.1809 & -0.9584 \\ -12.065 & 4.3910 & 1.2877 \\ 34.830 & 0.6572 & -3.4886 \end{bmatrix}$$

$$P_{n,2} = \begin{bmatrix} -0.0072 & 0.3017 & 0.2845 \\ 0.0009 & -0.0376 & -0.1761 \\ 0.0092 & -0.3725 & -1.8386 \end{bmatrix} \tag{85}$$

$$W_{n,3} = \begin{bmatrix} 11.665 & 2.2105 & -11.263 \\ -6.0256 & -1.1449 & -17.177 \\ 5.0019 & 0.9603 & -17.767 \end{bmatrix}$$

$$P_{n,3} = \begin{bmatrix} -0.0041 & 0.3916 & -0.9689 \\ 0.0005 & -0.0207 & -0.2238 \\ -0.0025 & 0.3185 & -1.8625 \end{bmatrix} \tag{86}$$

Note for all nominal filters the system matrix $A_e = A$. The robust (uncertainty explicitly considered) gain matrices obtained were

$$W_{r,1} = \begin{bmatrix} 0.0445 & 3.0049 & -0.0722 \\ -1.1029 & -0.7468 & 0.0282 \\ 12.8499 & 0.0867 & -0.1242 \end{bmatrix}$$

$$P_{r,1} = \begin{bmatrix} 0.0235 & 0.0430 & -0.2387 \\ -0.0072 & 0.2216 & 0.0108 \\ -0.0066 & -0.0337 & 0.0567 \end{bmatrix} \tag{87}$$

$$A_{e,1} = \begin{bmatrix} 0.9049 & 0.0109 & 0.0040 \\ 0.0004 & 0.7968 & 0.0025 \\ 3.789e{-6} & -0.0002 & 1.0687 \end{bmatrix} \tag{88}$$

$$W_{r,2} = \begin{bmatrix} -0.0218 & 0.5019 & -0.0161 \\ 0.1665 & -250.65 & 0.0066 \\ 6.7877 & 0.0019 & -0.0030 \end{bmatrix} \tag{89}$$

$$P_{r,2} = \begin{bmatrix} -4.143e{-7} & 0.2373 & -2.0956 \\ -4.423e{-7} & 0.0012 & 0.0062 \\ -2.894e{-6} & -8.729e{-5} & -0.0011 \end{bmatrix} \tag{90}$$

$$A_{e,2} = \begin{bmatrix} 0.5480 & 0.0021 & -0.0003 \\ -1.057e-5 & 0.8245 & -0.0063 \\ 9.35980069e-6 & -0.0013 & 2.4357 \end{bmatrix} \tag{91}$$

$$W_{r,3} = \begin{bmatrix} 9.1728 & 9.5942 & -1.8998 \\ -1.1983 & -3.0317 & -221.94 \\ -0.4734 & 0.7084 & -127.55 \end{bmatrix}$$

$$P_{r,3} = \begin{bmatrix} -0.0047 & 0.4759 & -0.3249 \\ -5.4336 & 0.0421 & -0.0067 \\ 0.0027 & -0.6124 & 1.36088 \end{bmatrix} \tag{92}$$

$$A_{e,3} = \begin{bmatrix} 0.4731 & 0.0192 & 0.0075 \\ 0.0035 & 0.0955 & 5.180e-5 \\ -3.203e-6 & -1.691e-5 & 0.2037 \end{bmatrix} \tag{93}$$

As has been shown, for both the nominal and robust systems three filters were designed corresponding to targeted faults $\boldsymbol{f}_1, \boldsymbol{f}_2$, and $\boldsymbol{f}_3$. To verify the solutions obtained by the RCGA, the frequency response of the closed-loop systems were examined. Specifically, Bode diagrams were used to check the magnitude of the transfer functions from the faults $\boldsymbol{f}_1, \boldsymbol{f}_2$, and $\boldsymbol{f}_3$ and the disturbances $\boldsymbol{w}_1$, $\boldsymbol{w}_2$, and $\boldsymbol{w}_3$ to the residual signals r_1, r_2, and r_3. Figure 4 shows the response of filter 3 of the nominal system, where the target fault is $\boldsymbol{f}_3$ and the residual signal is r_3. It can be seen that the influence of the target fault signal on the residual is significantly larger than the influence of the nuisance faults and disturbances over all frequencies. Similarly, in Fig. 5 filter 2 of the robust system, where the target fault is $\boldsymbol{f}_2$ and the residual signal is r_2, the influence of the target fault signal on the residual is larger than the nuisance faults and disturbance signals. (These trends are representative of the behavior of each filter response.)

To illustrate the application of the robust ℓ_1 estimator to robust fault detection, FDI of the system in model (75) and (76) subject to plant disturbances was performed. A bank of estimators (as described in Sec. III) was designed for the set of $\boldsymbol{y}_i$, $i \in \{1, 2, 3\}$, sensor outputs, that is, the ith estimator is designed to detect a fault in the $\boldsymbol{y}_i$ sensor while neglecting faults in the remaining sensors. Here the nominal case as well as the robust case are considered for the FDI process. Random white noise signals with zero mean were added as both the disturbance inputs and sensor noise. The variances of the disturbance inputs $\boldsymbol{w}_1$, $\boldsymbol{w}_2$, and $\boldsymbol{w}_3$ were 0.05, 0.08, and 0.03, respectively. To show the extent of robustness, uncertainty for all system matrices was considered. The uncertain parameters are assigned random values within their respective ranges. The values are given in Table 1.

This example only considered the occurrence of sensor faults within the system. A typical sensor fault in the jet engine is a drift

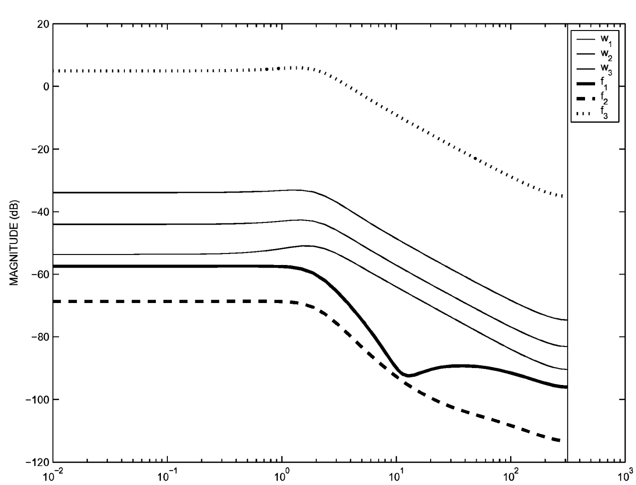


Fig. 4 Frequency response: nominal filter 3, target fault f_3 and residual signal r_3.

Table 1 Uncertain parameter values

Parameter	Fig. 6	Fig. 7	Fig. 8	Fig. 9
δ_{A_1}	0.004217	−0.000386	0.003415	−0.010360
δ_{A_2}	−0.011812	−0.007753	−0.009804	−0.007485
δ_{A_3}	0.007793	0.003931	0.006159	−0.000564
δ_{C_1}	0	−0.010086	−0.012755	−0.009738
δ_{C_2}	0	0.011859	0.018581	−0.001702
δ_{C_3}	0	−0.000169	0.001494	−0.004532

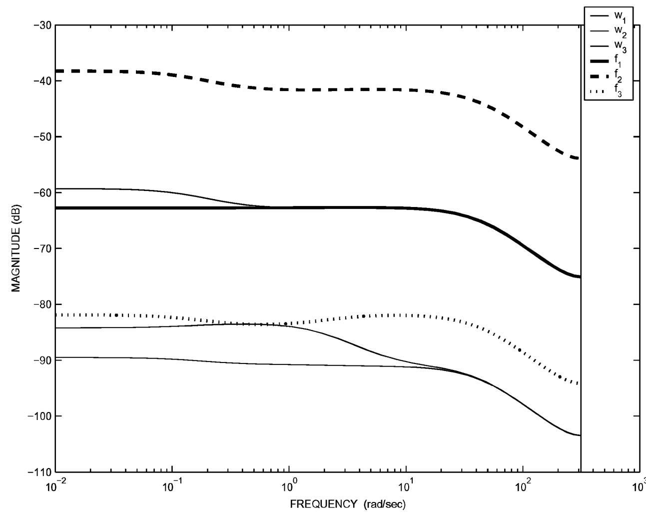


Fig. 5 Frequency response: robust filter 2, target fault f_2 and residual signal r_2, Bode diagram.

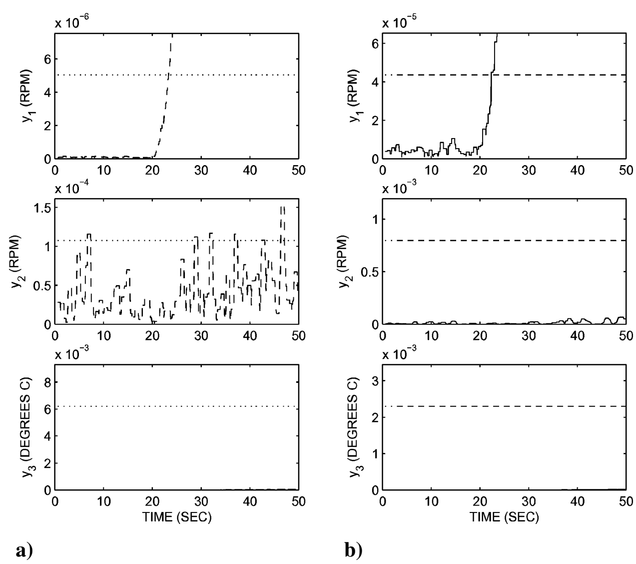


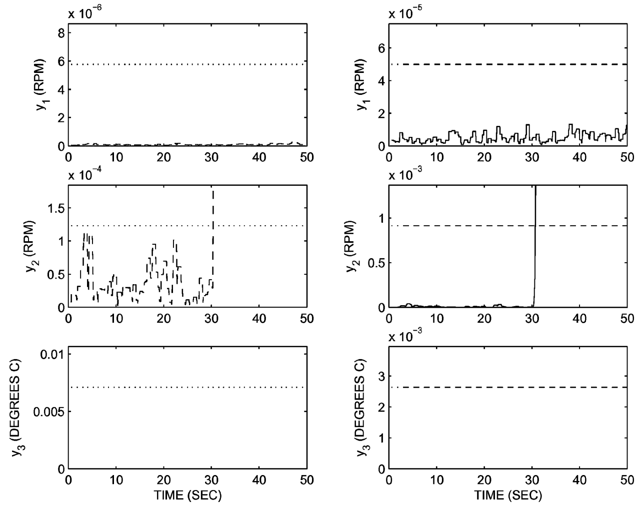
Fig. 6 Robust ℓ_1 FDI: fault in y_1 sensor at $t = 20$ s: a) nominal system and b) robust system.

in the sensor reading. Thus, a slow drifting (or ramping) sensor fault was added to a sensor reading at a particular instant in time. Specifically, the simulated fault signal can be described by the linear function

$$f_i(k) = \begin{cases} 0, & k < k_f \\ \tau(k - k_f), & k \geq k_f \end{cases} \tag{94}$$

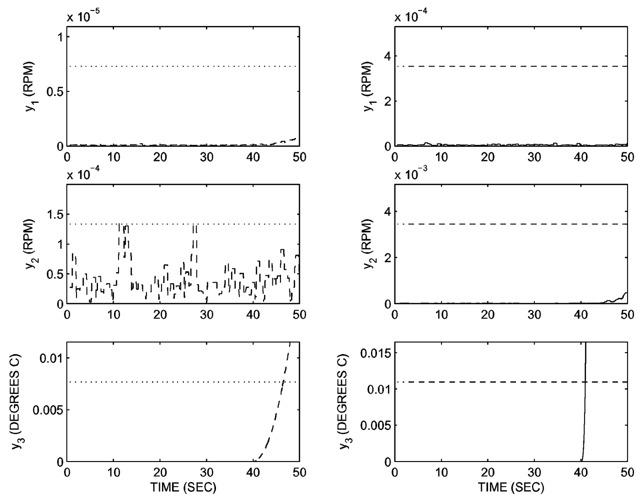
where $\tau = 0.1$ is the slope and k_f is the instant at which the fault occurs. Because of the disturbance, the finite-horizon infinity norm (64) of the residual with $N - N_0 = 60$ (corresponding to a time interval of 0.6 s) was nonzero even in the absence of faults.

In Figs. 6–8, a single sensor fault was introduced in the system. Specifically, in Fig. 6 a fault was introduced in sensor $\boldsymbol{y}_1$ at $t = 20$ s, Fig. 7 has a fault introduced in sensor $\boldsymbol{y}_2$ at $t = 30$ s, and a fault in



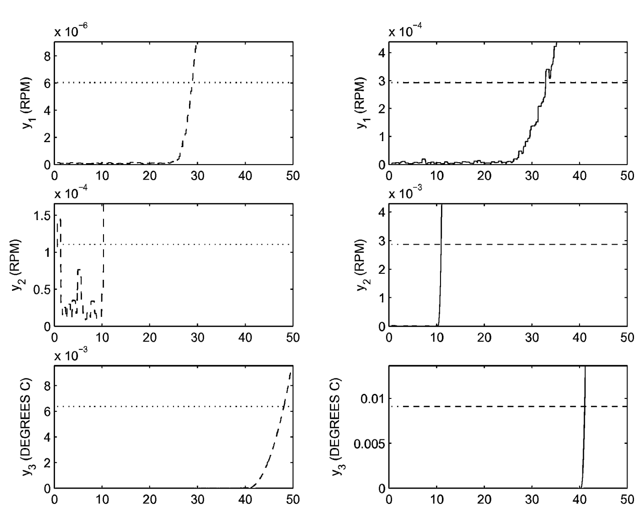
a) b)

Fig. 7 Robust ℓ_1 FDI: fault in y_2 sensor at $t = 30$ s: a) nominal system and b) robust system.



a) b)

Fig. 8 Robust ℓ_1 FDI: fault in y_3 sensor at $t = 40$ s: a) nominal system and b) robust system.



a) b)

Fig. 9 Robust ℓ_1 FDI: fault in y_1 at $t = 25$ s, y_2 at $t = 10$ s, and y_3 sensor at $t = 40$ s: a) nominal system and b) robust system.

sensor $\boldsymbol{y}_3$ is introduced at $t = 40$ s in Fig. 8. It can be seen that both the nominal and robust estimators were able to detect and isolate each fault successfully. It is evident at each faulty sensor that the residual surpassed its respective threshold at the the time of occurrence of each fault. However, note that in each nominal estimator system false alarms are given in one of the fault-free sensors. This is because uncertainty was not accounted for in the design of these estimators. These false alarms are avoided with the robust filters. In Fig. 9, multiple faults were introduced in sensors $\boldsymbol{y}_1$, $\boldsymbol{y}_2$, and $\boldsymbol{y}_3$ at $t = 25$ s, $t = 10$ s, and $t = 40$ s, respectively. Observe that in this instant both nominal and robust systems were able to isolate each target fault from the other nuisance faults. A false alarm is again given as the first residual surpasses its threshold well before a fault is introduced in the sensor. This does not occur with the robust estimator.

Conclusions

This paper considered the application of robust ℓ_1 estimation for uncertain, linear discrete-time systems to the robust FDI. MSSV theory of Ref. 18 was used to design a bank of robust ℓ_1 estimators and the resulting fixed threshold logic. When a discrete, linear model of a jet engine with real parametric uncertainties was considered and drifting sensor faults were introduced, it was shown that the robust FDI methodology based on fixed thresholds was capable of detecting and isolating failures in each of the particular sensors. Also, by the design of robust estimators to account for uncertainty explicitly, false alarm rates were significantly reduced.

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